Principles of Managerial Finance Solution
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CHAPTER 6

Interest Rates and Bond Valuation

INSTRUCTOR'S RESOURCES

Overview

This chapter begins with a thorough discussion of interest rates, yield curves, and their relationship to required returns. Features of the major types of bond issues are presented along with their legal issues, risk characteristics, and indenture convents. The chapter then introduces students to the important concept of valuation and demonstrates the impact of cash flows, timing, and risk on value. It explains models for valuing bonds and the calculation of yield-to-maturity using either the trial-and-error approach or the approximate yield formula.

PMF DISK

***PMF Tutor:* Bond and Stock Valuation**

This module provides problems for the valuation of conventional bonds and for the constant growth and variable growth models for common stock valuation.

***PMF Problem-Solver*: Bond and Stock Valuation**

This module's routines are Bond Valuation and Common Stock Valuation.

PMF Templates

Spreadsheet templates are provided for the following problems:

Problem	Topic
Self-Test 6-1	Bond valuation
Self-Test 6-2	Yield to maturity
Problem 6-2	Real rate of interest
Problem 6-24	Bond valuation–Semiannual interest
Problem 6-26	Bond valuation–Quarterly interest

Study Guide

Suggested *Study Guide* examples for classroom presentation:

Example	Topic
1	Valuation of any asset
4	Bond valuation
9	Yield to call

ANSWERS TO REVIEW QUESTIONS

6-1 The *real rate of interest* is the rate which creates an equilibrium between the supply of savings and demand for investment funds. The nominal rate of interest is the actual rate of interest charged by the supplier and paid by the demander. The nominal rate of interest differs from the real rate of interest due to two factors: (1) a premium due to inflationary expectations (IP) and (2) a premium due to issuer and issue characteristic risks (RP). The nominal rate of interest for a security can be defined as $k_1 = k^* + IP + RP$. For a three-month U.S. Treasury bill, the nominal rate of interest can be stated as $k_1 = k^* + IP$. The default risk premium, RP, is assumed to be zero since the security is backed by the U.S. government; this security is commonly considered the risk-free asset.

6-2 The *term structure of interest rates* is the relationship of the rate of return to the time to maturity for any class of similar-risk securities. The graphic presentation of this relationship is the yield curve.

6-3 For a given class of securities, the slope of the curve reflects an expectation about the movement of interest rates over time. The most commonly used class of securities is U.S. Treasury securities.

a. Downward sloping: long-term borrowing costs are lower than short-term borrowing costs.

b. Upward sloping: Short-term borrowing costs are lower than long-term borrowing costs.

c. Flat: Borrowing costs are relatively similar for short- and long-term loans.

The upward-sloping yield curve has been the most prevalent historically.

6-4 **a.** According to the *expectations theory*, the yield curve reflects investor expectations about future interest rates, with the differences based on inflation expectations. The curve can take any of the three forms. An upward-sloping curve is the result of increasing inflationary expectations, and vice versa.

b. The *liquidity preference theory* is an explanation for the upward-sloping yield curve. This theory states that long-term rates are generally higher than short-term rates due to the desire of investors for greater liquidity, and thus a premium must be offered to attract adequate long-term investment.

c. The *market segmentation theory* is another theory which can explain any of the three curve shapes. Since the market for loans can be segmented based on maturity, sources of supply and demand for loans within each segment determine the prevailing interest rate. If supply is greater than demand for short-term funds at a time when demand for long-term loans is higher than the supply of funding, the yield curve would be upward-sloping. Obviously, the reverse also holds true.

6-5 In the Fisher Equation, $k = k^* + IP + RP$, the risk premium, RP, consists of the following issuer- and issue-related components:

- *Default risk*. The possibility that the issuer will not pay the contractual interest or principal as scheduled.

- *Maturity (interest rate) risk*: The possibility that changes in the interest rates on similar securities will cause the value of the security to change by a greater amount the longer its maturity, and vice versa.

- *Liquidity risk*: The ease with which securities can be converted to cash without a loss in value.

- *Contractual provisions*: Covenants included in a debt agreement or stock issue defining the rights and restrictions of the issuer and the purchaser. These can increase or reduce the risk of a security.

- *Tax risk*: Certain securities issued by agencies of state and local governments are exempt from federal, and in some cases state and local, taxes, thereby reducing the nominal rate of interest by an amount which brings the return into line with the after-tax return on a taxable issue of similar risk.

The risks that are debt-specific are default, maturity, and contractual provisions.

6-6 Most corporate bonds are issued in denominations of $1,000 with maturities of 10 to 30 years. The *stated interest rate* on a bond represents the percentage of the bond's par value that will be paid out annually, although the actual payments may be divided up and made quarterly or semi-annually.

Both *bond indentures* and *trustees* are means of protecting the bondholders. The bond indenture is a complex and lengthy legal document stating the conditions under which a bond is issued. The trustee may be a paid individual, corporation, or commercial bank trust department that acts as a third-party "watch dog" on behalf of the bondholders to ensure that the issuer does not default on its contractual commitment to the bondholders.

6-7 Long-term lenders include *restrictive covenants* in loan agreements in order to place certain operating and/or financial constraints on the borrower. These constraints are intended to assure the lender that the borrowing firm will maintain a specified financial condition and managerial structure during the term of the loan. Since the lender is committing funds for a long period of time, he seeks to protect himself against adverse financial developments that may affect the borrower. The restrictive provisions (also called *negative covenants*) differ from the so-called *standard debt provisions* in that they place certain constraints on the firm's operations, whereas the standard provisions (also called *affirmative covenants*) require the firm to operate in a respectable and businesslike manner. Standard provisions include such requirements as providing audited financial statements on a regular schedule, paying taxes and liabilities when due, maintaining all facilities in good working order, and keeping accounting records in accordance with GAAP.

Violation of any of the standard or restrictive loan provisions gives the lender the right to demand immediate repayment of both accrued interest and principal of the loan. However, the lender does not normally demand immediate repayment but instead evaluates the situation in order to determine if the violation is serious enough to jeopardize the loan. The lender's options are: Waive the violation, waive the violation and renegotiate terms of the original agreement, or demand repayment.

6-8 *Short-term borrowing* is normally less expensive than *long-term borrowing* due to the greater uncertainty associated with longer maturity loans. The major factors affecting the cost of long-term debt (or the interest rate), in addition to loan maturity, are loan size, borrower risk, and the basic cost of money.

6-9 If a bond has a *conversion feature*, the bondholders have the option of converting the bond into a certain number of shares of stock within a certain period of time. A *call feature* gives the issuer the opportunity to repurchase, or call, bonds at a stated price prior to maturity. It provides extra compensation to bondholders for the potential opportunity losses that would result if the bond were called due to declining interest rates. This feature allows the issuer to retire outstanding debt prior to maturity and, in the case of convertibles, to force conversion. *Stock purchase warrants*, which are sometimes included as part of a bond issue, give the holder the right to purchase a certain number of shares of common stock at a specified price.

Bonds are rated by independent rating agencies such as Moody's and Standard & Poor's with respect to their overall quality, as measured by the safety of repayment of principal and interest. Ratings are the result of detailed financial ratio and cash flow analyses of the issuing firm. The bond rating affects the rate of return on the bond. The higher the rating, the less risk and the lower the rate.

6-10 The bond quotation for corporate bonds includes six pieces of information of interest to the investor. It includes the name of the issuer, the coupon rate, the year of maturity, the volume of bonds traded for the reporting day, the trading price for the last trade of the day (called the close price), and the change in the last trading price from the preceding trading day. The closing price and the change in price are quoted as a percent of the maturity value of the bond.

6-11 *Eurobonds* are bonds issued by an international borrower and sold to investors in countries with currencies other than that in which the bond is denominated. For example, a dollar-denominated Eurobond issued by an American corporation can be sold to French, German, Swiss, or Japanese investors. A *foreign bond*, on the other hand, is issued by a foreign borrower in a host country's capital market and denominated in the host currency. An example is a French-franc denominated bond issued in France by an English company.

6-12 A financial manager must understand the valuation process in order to judge the value of benefits received from stocks, bonds, and other assets in view of their risk, return, and combined impact on share value.

6-13 Three key inputs to the valuation process are:

1. *Cash flows* - the cash generated from ownership of the asset;
2. *Timing* - the time period(s) in which cash flows are received; and
3. *Required return* - the interest rate used to discount the future cash flows to a present value. The selection of the required return allows the level of risk to be adjusted; the higher the risk, the higher the required return (discount rate).

6-14 The valuation process applies to assets that provide an intermittent cash flow or even a single cash flow over any time period.

6-15 The value of any asset is the present value of future cash flows expected from the asset over the relevant time period. The three key inputs in the valuation process are cash flows, the required rate of return, and the timing of cash flows. The equation for value is:

$$V_0 = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \cdots\cdots \frac{CF_n}{(1+k)^n}$$

where:

V_0 = value of the asset at time zero
CF_I = cash flow expected at the end of year t
k = appropriate required return (discount rate)
n = relevant time period

6-16 The basic *bond valuation equation* for a bond that pays annual interest is:

$$V_0 = I \times \left[\sum_{t=1}^{n} \frac{1}{(1+k_d)^t}\right] + M \times \left[\frac{1}{(1+k_d)^n}\right]$$

where:

V_0	=	value of a bond that pays annual interest
I	=	interest
n	=	years to maturity
M	=	dollar par value
k_d	=	required return on the bond

To find the value of bonds paying interest semiannually, the basic bond valuation equation is adjusted as follows to account for the more frequent payment of interest:

1. The annual interest must be converted to semiannual interest by dividing by two.
2. The number of years to maturity must be multiplied by two.
3. The required return must be converted to a semiannual rate by dividing it by 2.

6-17 A bond sells at a *discount* when the required return exceeds the coupon rate. A bond sells at a *premium* when the required return is less than the coupon rate. A bond sells at par value when the required return equals the coupon rate. The coupon rate is generally a fixed rate of interest, whereas the required return fluctuates with shifts in the cost of long-term funds due to economic conditions and/or risk of the issuing firm. The disparity between the required rate and the coupon rate will cause the bond to be sold at a discount or premium.

6-18 If the required return on a bond is constant until maturity and different from the coupon interest rate, the bond's value approaches its $1,000 par value as the time to maturity declines.

6-19 To protect against the impact of rising interest rates, a risk-averse investor would prefer bonds with short periods until maturity. The responsiveness of the bond's market value to interest rate fluctuations is an increasing function of the time to maturity.

6-20 The *yield-to-maturity (YTM)* on a bond is the rate investors earn if they buy the bond at a specific price and hold it until maturity. The trial-and-error approach to calculating the YTM requires finding the value of the bond at various rates to determine the rate causing the calculated bond value to equal its current value. The approximate approach for calculating YTM uses the following equation:

$$\text{Approximate Yield} = \frac{I + [(M - B_0)/n]}{(M + B_0)/2}$$

where:

I	=	annual interest
M	=	maturity value
B_o	=	market value
n	=	periods to maturity

The YTM can be found precisely by using a hand-held financial calculator and using the time value functions. Enter the B_0 as the present value, the I as the annual payment, and the n as the number of periods until maturity. Have the calculator solve for the interest rate. This interest value is the YTM. Many calculators are already programmed to solve for the Internal Rate of Return (IRR). Using this feature will also obtain the YTM since the YTM and IRR are determined the same way.

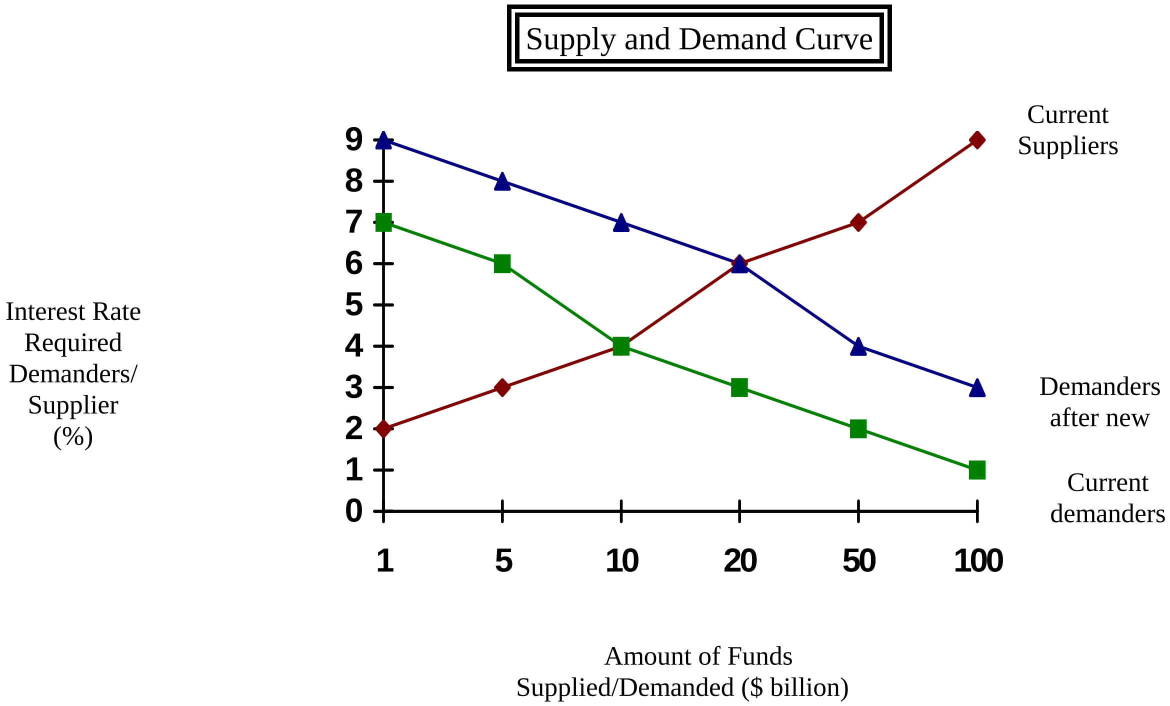
SOLUTIONS TO PROBLEMS

6-1 LG 1: Interest Rate Fundamentals: The Real Rate of Return

Real rate of return = 5.5% - 2.0% = 3.5%

6-2 LG 1: Real Rate of Interest

a.



b. The real rate of interest creates an equilibrium between the supply of savings and the demand for funds, which is shown on the graph as the intersection of lines for current suppliers and current demanders. K_0 = 4%

c. See graph.

d. A change in the tax law causes an upward shift in the demand curve, causing the equilibrium point between the supply curve and the demand curve (the real rate of interest) to rise from k_o = 4% to k_0 = 6% (intersection of lines for current suppliers and demanders after new law).

6-3 LG 1: Real and Nominal Rates of Interest

a. 4 shirts

b. $100 + ($100 x .09) = $109

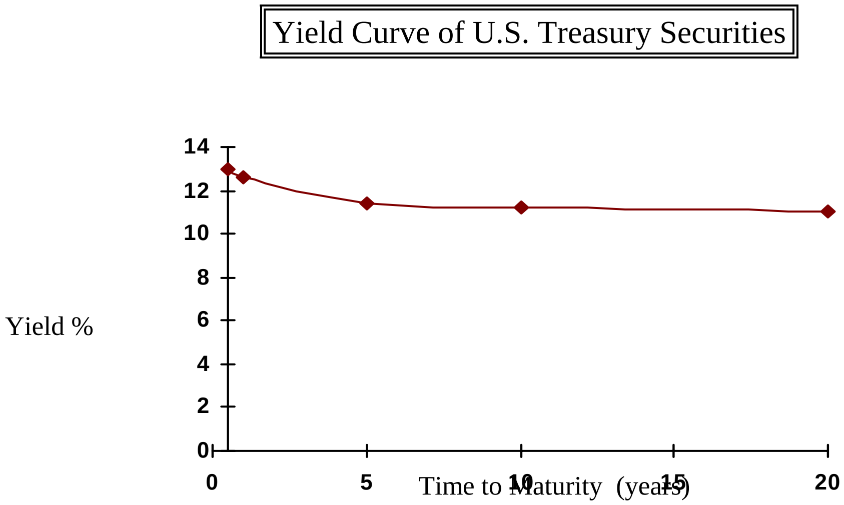
c. $25 + ($25 x .05) = $26.25

d. The number of polo shirts in one year = $109 ÷ $26.25 = 4.1524. He can buy 3.8% more shirts (4.1524 ÷ 4 = .0381).

e. The real rate of return is 9% - 5% = 4%. The change in the number of shirts that can be purchased is determined by the real rate of return since the portion of the nominal return for expected inflation (5%) is available just to maintain the ability to purchase the same number of shirts.

6-4 LG 1: Yield Curve

a.



b. The yield curve is slightly downward sloping, reflecting lower expected future rates of interest. The curve may reflect a general expectation for an economic recovery due to inflation coming under control and a stimulating impact on the economy from the lower rates.

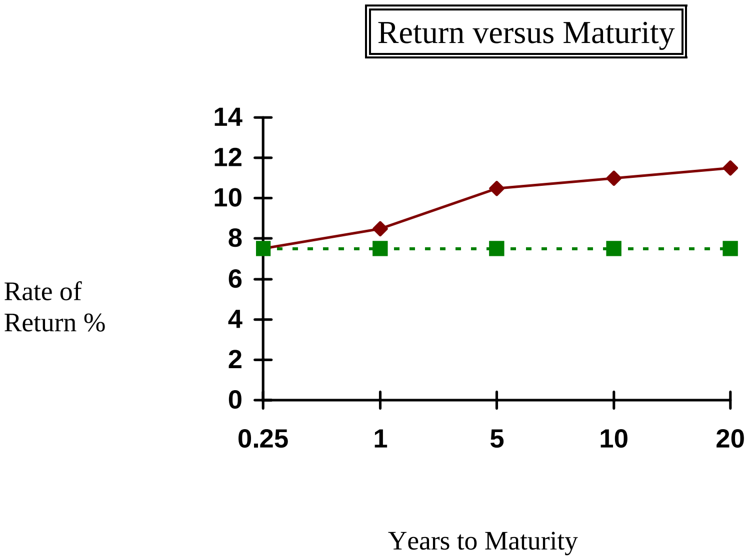
6-5 LG 1: Nominal Interest Rates and Yield Curves

a. $k_l = k^* + IP + RP_1$
For U.S. Treasury issues, RP = 0
$R_F = k^* + IP$

20 year bond:	R_F	=	2.5 + 9%	=	11.5%
3 month bill:	R_F	=	2.5 + 5%	=	7.5%
1 year note:	R_F	=	2.5 + 6%	=	8.5%
5 year bond:	R_F	=	2.5 + 8%	=	10.5%

b. If the real rate of interest (k*) drops to 2.0%, the nominal interest rate in each case would decrease by 0.5 percentage point.

c.



The yield curve for U.S. Treasury issues is upward sloping, reflecting the prevailing expectation of higher future inflation rates.

d. Followers of the liquidity preference theory would state that the upward sloping shape of the curve is due to the desire by lenders to lend short-term and the desire by business to borrow long term. The dashed line in the part **c** graph shows what the curve would look like without the existence of liquidity preference, ignoring the other yield curve theories.

e. Market segmentation theorists would argue that the upward slope is due to the fact that under current economic conditions there is greater demand for long-term loans for items such as real estate than for short-term loans such as seasonal needs.

6-6 LG 1: Nominal and Real Rates and Yield Curves

Real rate of interest (k*):

k_i = k* + IP + RP

RP = 0 for Treasury issues

k* = k_i - IP

a.

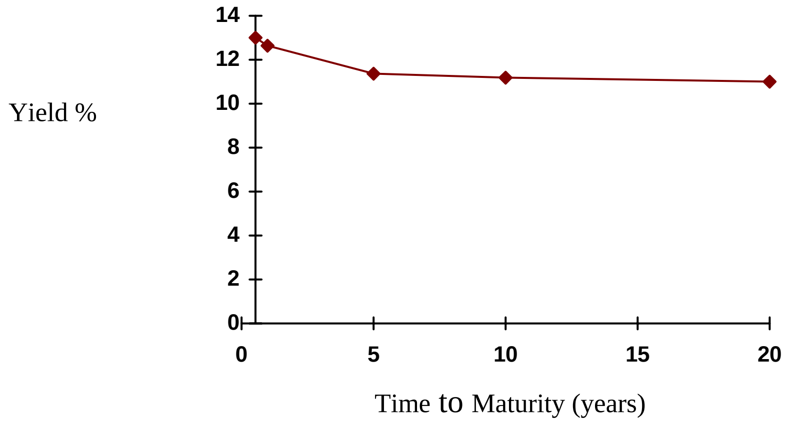
Security	Nominal rate (kj)	-	IP	=	Real rate of interest (k*)
A	12.6%	-	9.5%	=	3.1%

B	11.2%	-	8.2%	=	3.0%
C	13.0%	-	10.0%	=	3.0%
D	11.0%	-	8.1%	=	2.9%
E	11.4%	-	8.3%	=	3.1%

b. The real rate of interest decreased from January to March, remained stable from March through August, and finally increased in December. Forces which may be responsible for a change in the real rate of interest include changing economic conditions such as the international trade balance, a federal government budget deficit, or changes in tax legislation.

c.

Yield Curve of U.S. Treasury Securities

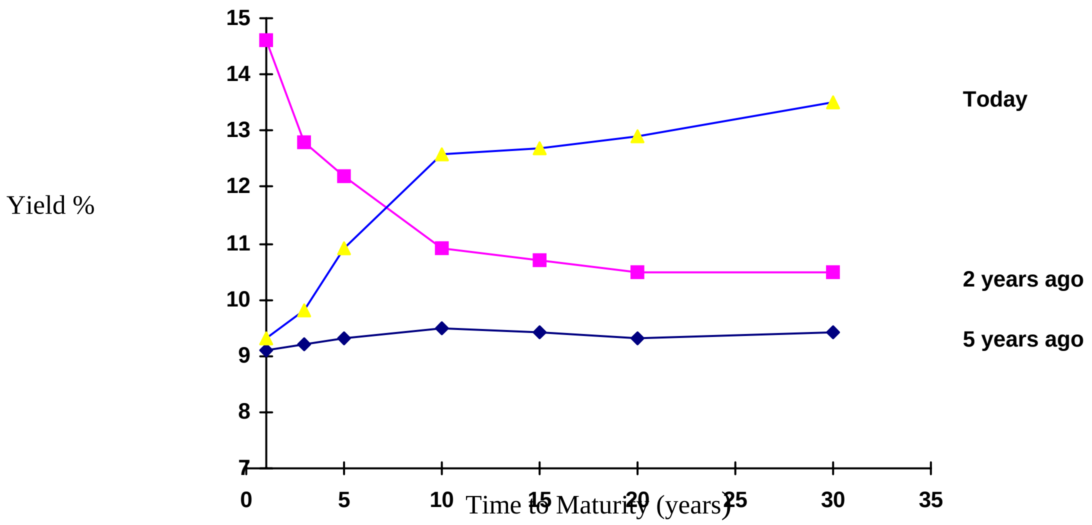


d. The yield curve is slightly downward sloping, reflecting lower expected future rates of interest. The curve may reflect a general expectation for an economic recovery due to inflation coming under control and a stimulating impact on the economy from the lower rates.

6-7 LG 1: Term Structure of Interest Rates

a.

Yield Curve of High-Quality Corporate Bonds



b. and c.

Five years ago, the yield curve was relatively flat, reflecting expectations of stable interest rates and stable inflation. Two years ago, the yield curve was downward sloping, reflecting lower expected interest rates due to a decline in the expected level of inflation. Today, the yield curve is upward sloping, reflecting higher expected inflation and higher future rates of interest.

6-8 LG 1: Risk-Free Rate and Risk Premiums

a. Risk-free rate: $R_F = k^* + IP$

Security	k*	+	IP	=	R_F
A	3%	+	6%	=	9%
B	3%	+	9%	=	12%
C	3%	+	8%	=	11%
D	3%	+	5%	=	8%
E	3%	+	11%	=	14%

b. Since the expected inflation rates differ, it is probable that the maturity of each security differs.

c. Nominal rate: $k = k^* + IP + RP$

Security	k*	+	IP	+	RP	=	k
A	3%	+	6%	+	3%	=	12%
B	3%	+	9%	+	2%	=	14%
C	3%	+	8%	+	2%	=	13%
D	3%	+	5%	+	4%	=	12%
E	3%	+	11%	+	1%	=	15%

6-9 LG 1: Risk Premiums

a. $R_{Ft} = k^* + IP_t$

Security A: R_{F3} = 2% + 9% = 11%

Security B: R_{F15} = 2% + 7% = 9%

b. Risk premium:
RP = default risk + interest rate risk + liquidity risk + other risk
Security A: RP = 1% + 0.5% + 1% + 0.5% = 3%
Security B: RP = 2% + 1.5% + 1% + 1.5% = 6%

c. $k_i = k^* + IP + RP$ or $k_1 = R_F$ + Risk premium
Security A: k_1 = 11% + 3% = 14%
Security B: k_1 = 9% + 6% = 15%

Security A has a higher risk-free rate of return than Security B due to expectations of higher near-term inflation rates. The issue characteristics of Security A in comparison to Security B indicate that Security A is less risky.

6-10 LG 2: Bond Interest Payments Before and After Taxes

a. Yearly interest = ($1,000 x .07) = $70.00

b. Total interest expense = $70.00 per bond x 2,500 bonds = $175,000

c.

Total before tax interest	$175,000
Interest expense tax savings (.35 x $175,000)	61,250
Net after-tax interest expense	$113,750

6-11 LG 3: Bond Quotation

a. Tuesday, November 7
b. 1.0025 x $1,000 = $1,002.50
c. 2005d
d. 558
e. 8 3/4%
f. current yield = $87.50 ÷ $1,002.50 = 8.73% or 8.7% per the quote
g. The price declined by 5/8% of par value. This decline means the previous close was 100 7/8 or $1,008.75.

6-12 LG 4: Valuation Fundamentals

a. Cash Flows: CF_{1-5} $1,200
CF_5 $5,000
Required return: 6%

b.
$$V_0 = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \frac{CF_3}{(1+k)^3} + \frac{CF_4}{(1+k)^4} + \frac{CF_5}{(1+k)^5}$$

$$V_0 = \frac{\$1,200}{(1+.06)^1} + \frac{\$1,200}{(1+06)^2} + \frac{\$1,200}{(1+06)^3} + \frac{\$1,200}{(1+06)^4} + \frac{\$6,200}{(1+06)^5}$$

$$V_0 = \$8,791$$

Using PVIF formula:
V_0 = [(CF$_1$ x PVIF$_{6\%,l}$) + (CF$_2$ x PVIF$_{6\%, 2}$) ... (CF$_5$ x PVIF$_{6\%,5}$)]
V_0 = [($1,200 x .943) + ($1,200 x .890) + ($1,200 x .840) + ($1,200 x .792) + ($6,200 x.747)]
V_0 = $1,131.60 + $1,068.00 + $1,008 + $950.40 + $4,631.40
V_0 = $8,789.40
Calculator solution: $8,791.13

The maximum price you should be willing to pay for the car is $8,789, since if you paid more than that amount, you would be receiving less than your required 6% return.

6-13 LG 4: Valuation of Assets

Asset	End of Year	Amount	PVIF or PVIFA$_{k\%,n}$	Present Value of Cash Flow
A	1	$5000		
	2	$5000	2.174	
	3	$5000		$10,870.00
			Calculator solution:	$10,871.36
B	1 - ∞	$ 300	1 ÷ .15	$ 2,000
C	1	0		
	2	0		
	3	0		
	4	0		
	5	$35,000	.476	$16,660.00
			Calculator solution:	$16,663.96
D	1-5	$1,500	3.605	$ 5,407.50
	6	8,500	.507	4,309.50
				$ 9,717.00
			Calculator solution:	$ 9,713.52
E	1	$2,000	.877	$ 1,754.00
	2	3,000	.769	2,307.00
	3	5,000	.675	3,375.00

4	7,000	.592	4,144.00
5	4,000	.519	2,076.00
6	1,000	.456	456.00
			$14,112.00
		Calculator solution:	$14,115.27

6-14 LG 1: Asset Valuation and Risk

a.

		10% Low Risk		15% Average Risk		22% High Risk	
		PVIFA	PV of CF	PVIFA	PV of CF	PVIFA	PV of CF
CF_{1-4}	$3,000	3.170	$ 9,510	2.855	$ 8,565	2.494	$ 7,482
CF_5	15,000	.621	9,315	.497	7,455	.370	5,550
Present Value of CF:			$18,825		$ 16,020		$13,032
Calculator solutions:			$18,823.42		$16,022.59		$13,030.91

b. The maximum price Laura should pay is $13,032. Unable to assess the risk, Laura would use the most conservative price, therefore assuming the highest risk.

c. By increasing the risk of receiving cash flow from an asset, the required rate of return increases, which reduces the value of the asset.

6-15 LG 5: Basic Bond Valuation

a.
$B_o = I \times (PVIFA_{kd\%,n}) + M \times (PVIF_{kd\%,n})$
$B_o = 120 \times (PVIFA_{10\%,16}) + M \times (PVIF_{10\%,16})$
$B_o = \$120 \times (7.824) + \$1,000 \times (.218)$
$B_o = \$938.88 + \$218$
$B_o = \$1,156.88$
Calculator solution: $1,156.47

b. Since Complex Systems' bonds were issued, there may have been a shift in the supply-demand relationship for money or a change in the risk of the firm.

c.
$B_o = I \times (PVIFA_{kd\%,n}) + M \times (PVIF_{kd\%,n})$
$B_o = 120 \times (PVIFA_{12\%,16}) + M \times (PVIF_{12\%,16})$
$B_o = \$120 \times (6.974) + \$1,000 \times (.163)$
$B_o = \$836.88 + \$163$
$B_o = \$999.88$
Calculator solution: $1,000

When the required return is equal to the coupon rate, the bond value is equal to the par value. In contrast to **a.** above, if the required return is less than the coupon rate, the bond will sell at a premium (its value will be greater than par).

6-16 LG 5: Bond Valuation–Annual Interest

$\mathbf{B_o = I \times (PVIFA_{kd\%,n}) + M \times (PVIF_{kd\%,n})}$

Calculator

Bond	Table Values			Solution
A	B_o = $140 x (7.469) + $1,000 x (.104)	=	$1,149.66	$1,149.39
B	B_o = $80 x (8.851) + $1,000 x (.292)	=	$1,000.00	$1,000.00
C	B_o = $10 x (4.799) + $100 x (.376)	=	$ 85.59	$ 85.60
D	B_o = $80 x (4.910) + $500 x (.116)	=	$ 450.80	$ 450.90
E	B_o = $120 x (6.145) + $1,000 x (.386)	=	$1,123.40	$1,122.89

6-17 LG 5: Bond Value and Changing Required Returns

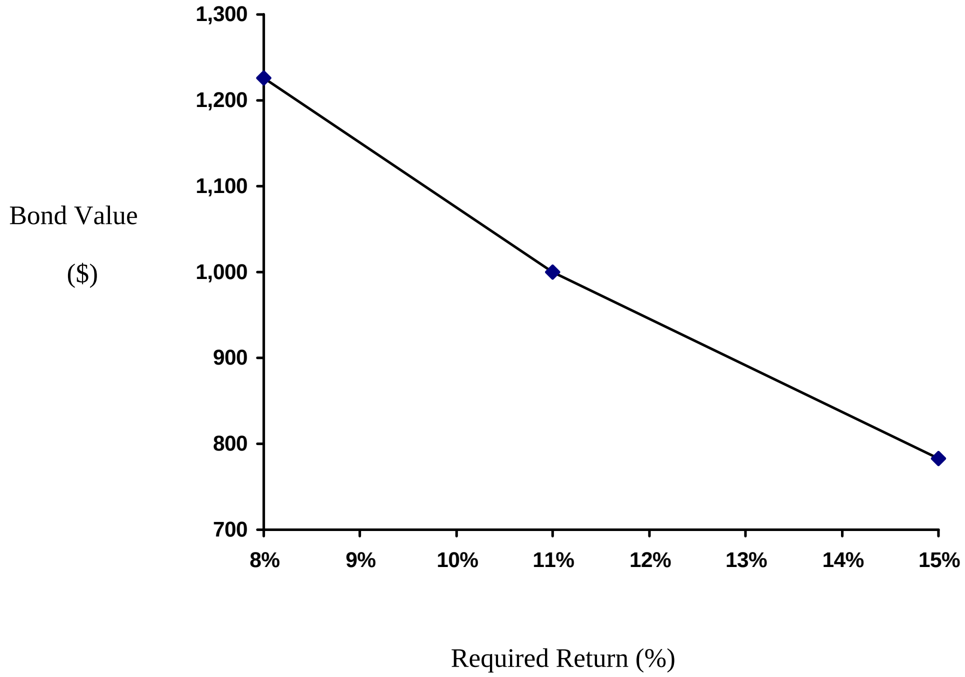
$B_o = I \times (PVIFA_{kd\%,n}) + M \times (PVIF_{kd\%,n})$

a.

Bond	Table Values			Calculator Solution
(1)	Bo = $110 x (6.492) + $1,000 x (.286)	=	$1,000.00	$1,000.00
(2)	Bo = $110 x (5.421) + $1,000 x (.187)	=	$ 783.31	$ 783.18
(3)	Bo = $110 x (7.536) + $1,000 x (.397)	=	$1,225.96	$1,226.08

b.

Bond Value versus Required Return



c. When the required return is less than the coupon rate, the market value is greater than the par value and the bond sells at a premium. When the required return is greater than the coupon rate, the market value is less than the par value; the bond therefore sells at a discount.

d. The required return on the bond is likely to differ from the coupon interest rate because either (1) economic conditions have changed, causing a shift in the basic cost of long-term funds, or (2) the firm's risk has changed.

6-18 LG 5: Bond Value and Time–Constant Required Returns

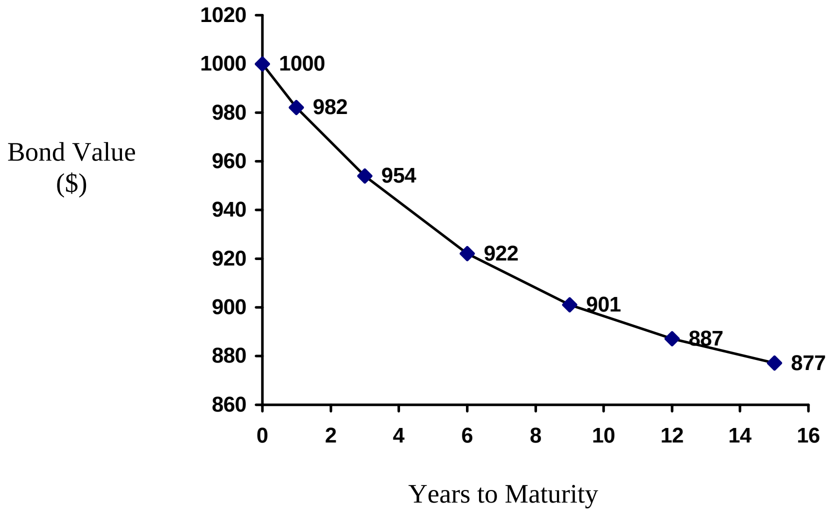
$B_o = I \times (PVIFA_{kd\%,n}) + M \times (PVIF_{kd\%,n})$

a.

Bond	Table Values		Calculator Solution
(1)	Bo = \$120 x (6.142) + \$1,000 x (.140) =	\$ 877.04	\$ 877.16
(2)	Bo = \$120 x (5.660) + \$1,000 x (.208) =	\$ 887.20	\$ 886.79
(3)	Bo = \$120 x (4.946) + \$1,000 x (.308) =	\$ 901.52	\$ 901.07
(4)	Bo = \$120 x (3.889) + \$1,000 x (.456) =	\$ 922.68	\$ 922.23
(5)	Bo = \$120 x (2.322) + \$1,000 x (.675) =	\$ 953.64	\$ 953.57
(6)	Bo = \$120 x (0.877) + \$1,000 x (.877) =	\$ 982.24	\$ 982.46

b.

Bond Value versus Years to Maturity



c. The bond value approaches the par value.

6-19 LG 5: Bond Value and Time–Changing Required Returns

B_o = I x $(PVIFA_{kd\%,n})$ + M x $(PVIF_{kd\%,n})$

a.

Bond	Table Values		Calculator Solution
(1)	B_0 = \$110 x (3.993) + \$1,000 x (.681) =	\$1,120.23	\$1,119.78
(2)	B_0 \$110 x (3.696) + \$1,000 x (.593) =	\$1,000.00	\$1,000.00
(3)	B_0 = \$110 x (3.433) + \$1,000 x (.519) =	\$ 896.63	\$ 897.01

b.

Bond	Table Values		Calculator Solution
(1)	B_0 = \$110 x (8.560) + \$1,000 x (.315) =	\$1,256.60	\$1,256.78
(2)	B_0 \$110 x (7.191) + \$1,000 x (.209) =	\$1,000.00	\$1,000.00
(3)	B_0 = \$110 x (6.142) + \$1,000 x (.140) =	\$ 815.62	\$ 815.73

c.

	Value	
Required Return	Bond A	Bond B
8%	\$1,120.23	\$1,256.60
11%	1,000.00	1,000.00
14%	896.63	815.62

The greater the length of time to maturity, the more responsive the market value of the bond to changing required returns, and vice versa.

d. If Lynn wants to minimize interest rate risk in the future, she would choose Bond A with the shorter maturity. Any change in interest rates will impact the market value of Bond A less than if she held Bond B.

6-20 LG 6: Yield to Maturity

Bond A is selling at a discount to par.
Bond B is selling at par value.
Bond C is selling at a premium to par.

Bond D is selling at a discount to par.
Bond E is selling at a premium to par.

6-21 LG 6: Yield to Maturity

a. Using a financial calculator the YTM is 12.685%. The correctness of this number is proven by putting the YTM in the bond valuation model. This proof is as follows:

B_o = 120 x ($PVIFA_{12.685\%,15}$) + 1,000 X ($PVIF_{12.685\%,15}$)
B_o = \$120 x (6.569) + \$1,000 x (.167)
B_o = \$788.28 + 167
B_o = \$955.28

Since B_0 is \$955.28 and the market value of the bond is \$955, the YTM is equal to the rate derived on the financial calculator.

b. The market value of the bond approaches its par value as the time to maturity declines. The yield to maturity approaches the coupon interest rate as the time to maturity declines.

6-22 LG 6: Yield to Maturity

a.

Bond	Approximate YTM	Trial-and-error YTM Approach	Error (%)	Calculator Solution
A	$= \frac{\$90 + [(\$1{,}000 - \$820) \div 8]}{[(\$1{,}000 + \$820) \div 2]}$			
	= 12.36%	12.71%	-0.35	12.71%
B	= 12.00%	12.00%	0.00	12.00%
C	$= \frac{\$60 + [(\$500 - \$560) \div 12]}{[(\$500 + \$560) \div 2]}$			
	= 10.38%	10.22%	+0.15	10.22%

Bond	Approximate YTM	Trial-and-error YTM Approach	Error (%)	Calculator Solution
D	$= \frac{\$150 + [(\$1{,}000 - \$1{,}120) \div 10]}{[(\$1{,}000 + \$1{,}120 \div 2]}$			
	= 13.02%	12.81%	+0.21	12.81%
E	$= \frac{\$50 + [(\$1{,}000 - \$900) \div 3]}{[(\$1{,}000 + \$900) \div 2]}$			
	= 8.77%	8.94%	-.017	8.95%

b. The market value of the bond approaches its par value as the time to maturity declines. The yield-to-maturity approaches the coupon interest rate as the time to maturity declines.

6-23 LG 2, 5, 6: Bond Valuation and Yield to Maturity

a. B_A = \$60($PVIFA_{12\%,5}$) + \$1,000($PVIF_{12\%,5}$)
B_A = \$60(3.605) + \$1,000(.567)
B_A = \$216.30 + 567
B_A = \$783.30

B_B = \$140($PVIFA_{12\%,5}$) + \$1,000($PVIF_{12\%,5}$)
B_B = \$140(3.605) + \$1,000(.567)
B_B = \$504.70 + 567
B_B = \$1,071.70

b.

$$\text{Number of bonds} = \frac{\$20{,}000}{\$783.30} = 25.533 \text{ of bond A}$$

$$\text{Number of bonds} = \frac{\$20{,}000}{\$1{,}071.70} = 18.662 \text{ of bond B}$$

c. Interest income of A = 25.533 bonds x \$60 = \$1,531.98
Interest income of B = 18.66194 bonds x \$140 = \$2,612.67

d. At the end of the 5 years both bonds mature and will sell for par of \$1,000.

FV_A = \$60($FVIFA_{10\%,5}$) + \$1,000
FV_A = \$60(6.105) + \$1,000
FV_A = \$366.30 + \$1,000 = \$1,366.30

FV_B = \$140($FVIFA_{10\%,5}$) + \$1,000
FV_B = \$140(6.105) + \$1,000
FV_B = \$854.70 + \$1,000 = \$1,854.70

e. The difference is due to the differences in interest payments received each year. The principal payments at maturity will be the same for both bonds. Using the calculator, the yield to maturity of bond A is 11.77% and the yield to maturity of bond B is 11.59% with the 10% reinvestment rate for the interest payments. Mark would be better off investing in bond A. The reasoning behind this result is that for both bonds the principal is priced to yield the YTM of 12%. However, bond B is more dependent upon the reinvestment of the large coupon payment at the YTM to earn the 12% than is the lower coupon payment of A.

6-24 LG 6: Bond Valuation–Semiannual Interest

B_o = I x ($PVIFA_{kd\%,n}$) + M x ($PVIF_{kd\%,n}$)
B_o = \$50 x ($PVIFA_{7\%,12}$) + M x ($PVIF_{7\%,12}$)
B_o = \$50 x (7.943) + \$1,000 x (.444)
B_o = \$397.15 + \$444
B_o = \$841.15
Calculator solution: \$841.15

6-25 LG 6: Bond Valuation–Semiannual Interest

B_o = I x ($PVIFA_{kd\%,n}$) + M x ($PVIF_{kd\%,n}$)

Bond	Table Values			Calculator Solution
A	B_o = \$50 x (15.247) + \$1,000 x (.390)	=	\$1,152.35	\$ 1,152.47
B	B_o = \$60 x (15.046) + \$1,000 x (.097)	=	\$1,000.00	\$ 1,000.00
C	B_o = \$30 x (7.024) + \$500 x (.508)	=	\$ 464.72	\$ 464.88
D	B_o = \$70 x (12.462) + \$1,000 x (.377)	=	\$1,249.34	\$ 1,249.24
E	B_o = \$3 x (5.971) + \$100 x (.582)	=	\$ 76.11	\$76.11

6-26 LG 6: Bond Valuation–Quarterly Interest

B_o = I x ($PVIFA_{kd\%,n}$) + M x ($PVIF_{kd\%,n}$)
B_o = \$125 x ($PVIFA_{3\%,40}$) + \$5,000 x ($PVIF_{3\%,40}$)
B_o = \$125 x (23.115) + \$5,000 x (.307)
B_o = \$2,889.38 + \$1,535
B_o = \$4,424.38
Calculator solution: \$4,422.13

CHAPTER 6 CASE
Evaluating Annie Hegg's Proposed Investment in Atilier Industries Bonds

This case demonstrates how a risky investment can affect a firm's value. First, students must calculate the current value of Suarez's bonds and stock, rework the calculations assuming that the firm makes the risky investment, and then draw some conclusions about the value of the firm in this situation. In addition to gaining experience in valuation of bonds and stock, students will see the relationship between risk and valuation.

a. Annie should convert the bonds. The value of the stock if the bond is converted is:

50 shares x $30 per share = $1,500

while if the bond was allowed to be called in the value would be on $1,080

b Current value of bond under different required returns – annual interest

(1) B_o = I x ($PVIFA_{6\%,25\ yrs.}$) + M x ($PVIF_{6\%,25\ yrs.}$)
B_o = $80 x (12.783) + $1,000 x (.233)
B_o = $1,022.64 + $233
B_o = $1,255.64
Ca1culator solution: $1,255.67
The bond would be at a premium.

(2) B_o = I x ($PVIFA_{8\%,25\ yrs.}$) + M x ($PVIF_{8\%,25\ yrs.}$)
B_o = $80 x (10.674) + $1,000 x (.146)
B_o = $853.92 + $146
B_o = $999.92
Ca1culator solution: $1,000.00
The bond would be at par value..

(3) B_o = I x ($PVIFA_{10\%,25\ yrs.}$) + M x ($PVIF_{10\%,25\ yrs.}$)
B_o = $80 x (9.077) + $1,000 x (.092)
B_o = $726.16 + $92
B_o = $818.16
Ca1culator solution: $818.46
The bond would be at a discount.

c Current value of bond under different required returns – semiannual interest

(1) B_o = I x ($PVIFA_{3\%,50\ yrs.}$) + M x ($PVIF_{3\%,50\ yrs.}$)
B_o = $40 x (25.730) + $1,000 x (.228)
B_o = $1,029.20 + $228

B_o = \$1,257.20
Ca1culator solution: \$1,257.30
The bond would be at a premium.

(2) B_o = I x ($PVIFA_{4\%,50\ yrs.}$) + M x ($PVI_{4\%,50\ yrs.}$)
B_o = \$40 x (21.482) + \$1,000 x (.141)
B_o = \$859.28 + \$146
B_o = \$1005.28
Ca1culator solution: \$1,000.00
The bond would be at par value..

(3) B_o = I x ($PVIFA_{5\%,50\ yrs.}$) + M x ($PVIF_{5\%,50\ yrs.}$)
B_o = \$40 x (18.256) + \$1,000 x (.087)
B_o = \$730.24 + \$87
B_o = \$817.24
Ca1culator solution: \$817.44
The bond would be at a discount.

Under all 3 required returns for both annual and semiannual interest payments the bonds are consistent in their direction of pricing. When the required return is above (below) the coupon the bond sells at a discount (premium). When the required return and coupon are equal the bond sells at par. When the change is made from annual to semiannual payments the value of the premium and par value bonds increase while the value of the discount bond decreases. This difference is due to the higher effective return associated with compounding frequency more often than annual.

d. If expected inflation increases by 1% the required return will increase from 8% to 9%, and the bond price would drop to \$908.84. This amount is the maximum Annie should pay for the bond.

B_o = I x ($PVIFA_{9\%,25\ yrs.}$) + M x ($PVIF_{9\%,25\ yrs.}$)
B_o = \$80 x (9.823) + \$1,000 x (.116)
B_o = \$785.84 + \$123
B_o = \$908.84
Ca1culator solution: \$901.77

e. The value of the bond would decline to \$925.00 due to the higher required return and the inverse relationship between bond yields and bond values.

B_o = I x ($PVIFA_{8.75\%,25\ yrs.}$) + M x ($PVIF_{8.75\%,25\ yrs.}$)
B_o = \$80 x (10.025) + \$1,000 x (.123)
B_o = \$802.00 + \$123
B_o = \$925.00
Ca1culator solution: \$924.81

f. The bond would increase in value and a gain of \$110.88 would be earned by Annie.

Bond value at 7% and 22 years to maturity.

B_o = I x ($PVIFA_{7\%,22\ yrs.}$) + M x ($PVIF_{7\%,22\ yrs.}$)
B_o = \$80 x (11.061) + \$1,000 x (.226)
B_o = \$884.88 + \$226
B_o = \$1,110.88
Ca1culator solution: \$1,110.61

g. The bond would increase in value and a gain of \$90.64 would be earned by Annie.

Bond value at 7% and 15 years to maturity.

B_o = I x ($PVIFA_{7\%,15\ yrs.}$) + M x ($PVIF_{7\%,15\ yrs.}$)
B_o = \$80 x (9.108) + \$1,000 x (.362)
B_o = \$728.64 + \$362
B_o = \$1,090.64
Ca1culator solution: \$1,091.08

The bond is more sensitive to interest rate changes when the time to maturity is longer (22 years) than when the time to maturity is shorter (15 years). Maturity risk decreases as the bond gets closer to maturity.

h. Using the calculator the YTM on this bond assuming annual interest payments of \$80, 25 years to maturity, and a current price of \$983.75 would be 8.15%.

i. Annie should probably not invest in the Atilier bond. There are several reasons for this conclusion.

1. The term to maturity is long and thus the maturity risk is high.
2. An increase in interest rates is likely due to the potential downgrading of the bond thus driving the price down.
3. An increase in interest rates is likely due to the possibility of higher inflation thus driving the price down.
4. The price of \$983.75 is well above her minimum price of \$908.84 assuming an increase in interest rates of 1%.